

Homework 2 for 8472 Spring 2012.

Due at 3:00pm on April 9th. No late homework will be accepted. Please turn in in class only in case I don't receive your homework. That is, do not put your homework in my mailbox or slip it into my office in case I don't get it. Please remember to write your name(s) legibly and to write the course number on the top of the first page.

In the following regression, we assume that Assumptions MLR1-6 hold.

1. Let  $wage$  denote dollars per hours,  $educ$  denotes years of schooling,  $exper$  the working experience on job, and  $tenure$  years on this job. And have the following results:

$$\widehat{wage} = -2.872 + 0.6 \cdot educ + 0.022 \cdot exper + 0.169 \cdot tenure \quad (1)$$

with  $n=526$ ,  $SST=7160$  and  $SSR=4966$ .

The covariance matrix of the estimators are the following:

$$\widehat{Var}(\hat{\beta}) = \begin{pmatrix} 0.5314 & -0.0357 & -.0042 & .0014 \\ & .00263 & .0002 & -.0001 \\ & & .00015 & -.00013 \\ & & & .0005 \end{pmatrix},$$
$$\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_{educ}, \hat{\beta}_{exper}, \hat{\beta}_{tenure})'. \quad (2)$$

- (a) Interpret the coefficient 0.6 on  $educ$ , 0.022 on  $exper$  and 0.169 on  $tenure$ .
- (b) What is the expected hourly wage for a worker with 12 years of education, 18 years of working experience and 10 years on this job.
- (c) Compute the unbiased estimator for  $\sigma^2$ . Compute the  $\widehat{sd}(\hat{\beta}_{educ})$ ,  $\widehat{sd}(\hat{\beta}_{exper})$ ,  $\widehat{sd}(\hat{\beta}_{tenure})$ ,  $\widehat{sd}(\hat{\beta}_{educ} + \hat{\beta}_{exper})$ , and  $\widehat{sd}(\hat{\beta}_{exper} - \hat{\beta}_{tenure})$ .
- (d) Compute the  $R^2$ .
- (e) Define the significance level of a test.
- (f) Test if  $\beta_{educ}$  is significantly different from 0 at 5% significance level. Construct the 2-sided 95% confidence interval of  $\beta_{educ}$ .
- (g) Test if  $exper$  and  $tenure$  have the same impact on wage at 10% significance level. Construct the 95% confidence interval of  $\beta_{exper} - \beta_{tenure}$ .

- (h) Test if *tenure* has a positive impact on wage at 5% significance level.
- (i) Construct a CI for  $\beta_{educ}$  such that you can make a statement that I am 90% confident that  $\beta_{educ}$  is greater than or equal to a specific number.
- (j) Construct a CI for  $\beta_{exper}$  such that you can make a statement that I am 95% confident that  $\beta_{exper}$  is less than or equal to a specific number.
- (k) Use the *F*-test to test the overall significance of the model at 10% significance level.
- (l) Use the Wald test to test  $H_0 : \beta_{educ} = \beta_{exper} = \beta_{tenure} = 0$  at 10% significance level.
- (m) Use the Wald test to test  $H_0 : \beta_{educ} = 0$  and  $\beta_{exper} + \beta_{tenure} = 2$  at 10% significance level.

2. Following 1, I also run the following regression:

$$\widehat{wage} = -0.904 + 0.54 \cdot educ, \quad (3)$$

and  $SSR = 5980$ .

- (a) What are the SST and SSE in this model?
  - (b) Compute the  $R^2$  for this model.
  - (c) use the F-test to test if  $H_0 : \beta_{exper} = \beta_{tenure} = 0$  at 5% significance level.
  - (d) Use the Wald test to test the same null hypothesis as in the previous problem. Do you get the same result?
3. Let  $y = \mathbf{x}\beta + u$  satisfy MLR1-MLR6 and  $\hat{\beta}$  the OLS estimator. Show that  $E[\hat{\beta}] = \beta$  and  $Var(\hat{\beta}) = \sigma^2 E[(\mathbf{X}'\mathbf{X})^{-1}]$ . Note that these are unconditional expectation and covariance matrix.