Online Supplement to "Analyzing the Performance of Multi-Factor Investment Strategies under Multiple Testing Framework"

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Appendix A

This appendix presents the Step-SPA(k) and FDP-SPA procedure to test the multiple inequalities

$$H^i: \theta_i \leq 0, \qquad i = 1, \dots, M.$$

Let *max*(*A*, *k*) and $1(\cdot)$ denote the k-th largest value of vector A and the indicator function, respectively; X_i denotes the vector of excess returns of portfolio i; and Y denotes the matrix of risk factor portfolios or the benchmark portfolio. There are two versions of test statistics that could be used: non-studentized and studentized. In the portfolio evaluation using alphas, we adopt the studentized test statistics, i.e. the *t*-ratio, so that they are comparable on the same scale. However, we use non-studentized test statistics when examining the Sharpe ratio differences.

The recentering estimator $\hat{\theta}_i^-$ is used to enhance the power of the test. Hansen [2005] shows that only the set of models or portfolios with $\theta_i = 0$ would affect the critical value of the k-th largest θ. Furthermore, the statistical power of multiple inequality testing could be substantially reduced if too many "irrelevant" inferior models are included. That is, if we can determine which $\theta_i < 0$, then we increase the power while maintaining the control of *k*-FWER. Hansen [2005] recommends the use of threshold $-\hat{\sigma}_i\sqrt{2\log\log T}$, which is based on law of iterated logarithm, for $\sqrt{T}\hat{\theta}_i$.

Step-SPA(k) algorithm with level δ

procedure stepSPA($\{X_1, \dots, X_M, Y\}, \delta, k$)

create vector STAT of size M

for $i \in \{1, \dots, M\}$ **do**

    calculate the parameter of interest $\hat{\theta}_i$ and its standard error $\hat{\sigma}_i$

$\hat{\theta}_i^- = \hat{\theta}_i \times 1(\sqrt{T}\hat{\theta}_i \leq -\hat{\sigma}_i\sqrt{2\log\log T})$

STAT[i] = $\sqrt{T}\hat{\theta}_i$ or $\sqrt{T}\hat{\theta}_i/\hat{\sigma}_i$ ◆ non-studentized or studentized test statistics

end for

create matrix X with row size M and column size B

for $s \in \{1, \dots, B\}$ **do**

generate bootstrap sample $\{X_1^s, \dots, X_M^s, Y^s\}$

for $i \in \{1, \dots, M\}$ **do**

compute $\hat{\theta}_i^s$ with the bootstrap sample

X[i, s] = $\sqrt{T}(\hat{\theta}_i^s - \hat{\theta}_i + \hat{\theta}_i^-)$ or $\sqrt{T}(\hat{\theta}_i^s - \hat{\theta}_i + \hat{\theta}_i^-)/\hat{\sigma}_i$ ◆ non-studentized or studentized test statistics

end for

end for

create sort_index which order the vector STAT from high to low

SORTED_X = X[sort_index, :] ◆ re-order rows of X according to sort_index

NUM_REJECT = 0

NUM_REJECT1 = -1

create vector KMAX of size B

while NUM_REJECT > NUM_REJECT1 **do** ◆ The procedure will stop when there is no further rejection

NUM_REJECT1 = NUM_REJECT

if NUM_REJECT < k **then do**

for $s \in \{1, \dots, B\}$ **do**

KMAX[s] = max(SORTED_X[: , s], k)

end for

else do

for $s \in \{1, \dots, B\}$ **do**

KMAX[s] = max(SORTED_X[(NUM_REJECT$-k+2$):M, s], k)

end for

end if

q = max(KMAX, $round(\delta \times B)$)

if q < 0 **then** q = 0 **end if**

CRITCAL_VALUE = q

NUM_REJECT = sum(1(STAT > CRITICAL_VALUE))

end while

Output: CRITICAL_VALUE

end procedure

FDP-SPA with α and ξ

procedure FDP_SPA($\{X_1, \dots, X_M, Y\}, \delta, \xi$)

k = 1

CRITICAL_VALUE = stepSPA($\{X_1, \dots, X_M, Y\}, \delta, k$)

NUM_REJECT = sum(1(STAT > CRITICAL_VALUE))

while NUM_REJECT < $k/\xi - 1$ **do**

    k = k + 1

    CRITICAL_VALUE = stepSPA($\{X_1, \dots, X_M, Y\}, \delta, k$)

    NUM_REJECT = sum(1(STAT > CRITICAL_VALUE))

end while

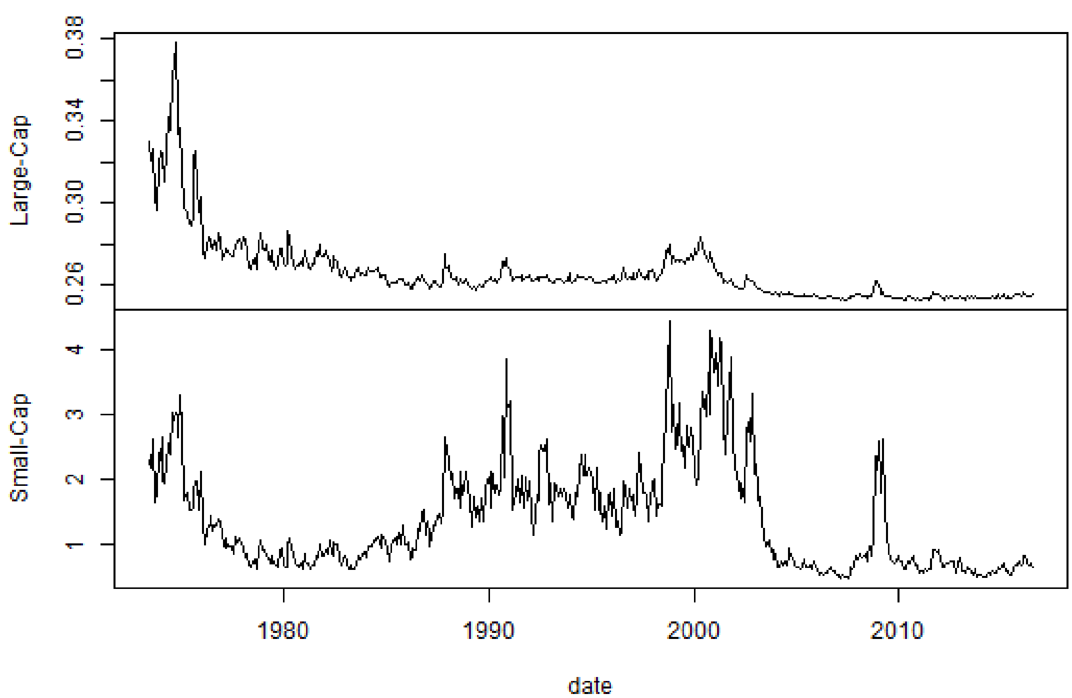
Output: CRITICAL_VALUE

end procedure

Appendix B

To demonstrate the difference in estimated transaction costs between trading large-cap and small-cap stocks sampled in this study, we provide time-series plots for the median estimates for both categories in Exhibit B1. We follow Fama and French [1993] to split the stocks into large-cap and small-cap groups with the NYSE median of market capitalization at the end of each month. For the large-cap stocks, we document that the transaction costs are trending downward in the past few decades. This result is consistent with the literature, see e.g., Chordia et al. [2014]. The median transaction cost is estimated to be between 0.25 and 0.3 for the large-cap stocks since 1976. The transaction costs are considerably greater for small-cap stocks, especially during the stock market downturn. In the era of crises, the high price impact explains the spike in the costs of trading small-cap stocks.

Exhibit B1. The Estimated Transaction Cost 1973-2016.



Note: The figures depict the monthly median transaction cost for large-cap and small-cap groups from June 1973 to June 2016. At the end of each month, all of the stocks are classified into large-cap and small-cap based on the NYSE median of market capitalization.

Reference

Chordia, T., A. Subrahmanyam, and Q. Tong. "Have Capital Market Anomalies Attenuated in the Recent Era of High Liquidity and Trading Activity?" *Journal of Accounting and Economics*, Vol. 58, No. 1 (2014), pp. 41-58.

Fama, E. F. and K. R. French. "Common Risk Factors in the Returns on Stocks and Bonds." *Journal of Financial Economics*, Vol. 33, No. 1 (1993), pp. 3-56.

Hansen, P. R. "A Test for Superior Predictive Ability." *Journal of Business & Economic Statistics*, Vol. 23, No. 4 (2005), pp. 365-380.